

OCR

Oxford Cambridge and RSA

Friday 22 June 2018 – Morning

A2 GCE MATHEMATICS (MEI)

4754/01 Applications of Advanced Mathematics (C4)

INSTRUCTIONS



The examination is in two parts:

Paper A (1 hour 30 minutes)

Paper B (up to 1 hour)

Supervisors are requested to ensure that Paper B **is not issued** until Paper A has been collected in from the candidates.

Centres may, if they wish, grant a supervised break between the two parts of this examination.

Paper B should not be attached to the corresponding Paper A script. For Paper A only the candidates' Printed Answer Books, in the same order as the attendance register, should be sent for marking; the Question Paper should be retained in the centre or recycled. For Paper B only the Question Papers, on which the candidates have written their answers, should be sent for marking; the Insert should be retained in the centre or recycled. Any additional sheets used must be carefully attached to the correct paper.

For Paper B (Comprehension) only.

A standard English dictionary is allowed for the comprehension.

(Dictionaries to be used in the examination must be thoroughly checked before the examination.) Full regulations are in the JCQ Regulations and Guidance booklet.

This notice must be on the invigilator's desk at all times during the morning of Friday 22 June 2018.

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Friday 22 June 2018 – Morning

A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

- 1 Express $\sin \theta - 2.4 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.
Hence write down the maximum value of the function $f(\theta) = 1 - \sin \theta + 2.4 \cos \theta$, where $0 \leq \theta \leq 2\pi$. [5]
- 2 The finite region bounded by the curve $y = \ln x$, the x -axis, the y -axis and the line $y = 1$ is rotated through 360° about the y -axis. Find the exact volume of the solid of revolution generated. [4]
- 3 Find the first three terms of the binomial expansion of $\frac{1+2x}{(2-x)^3}$ in ascending powers of x .
State the set of values of x for which the expansion is valid. [7]
- 4 A curve has parametric equations $x = \sin 2\theta$, $y = 1 + 2 \cos \theta - \cos 2\theta$, where $0 < \theta < \pi$.
(i) Find $\frac{dy}{dx}$ in terms of θ . [3]
(ii) Find the exact coordinates of the point on the curve where the gradient is zero. [4]
- 5 Fig. 5 shows the curve with equation $y = \sqrt{1+x^3}$.

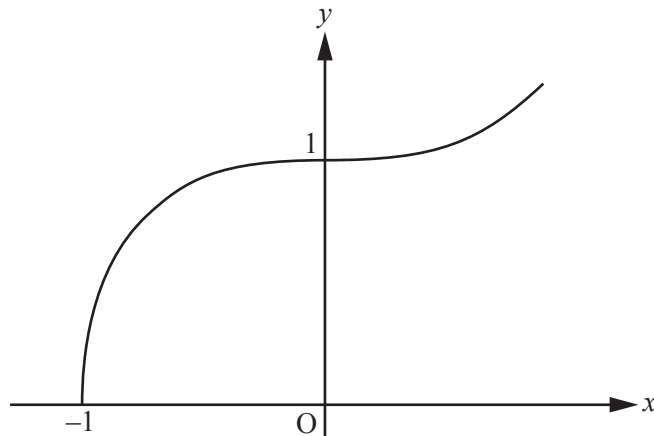


Fig. 5

- (i) Use the trapezium rule with 4 strips to estimate the finite area enclosed by the curve and the x - and y -axes, giving your answer correct to 3 significant figures. [3]
- (ii) Use a quarter circle of radius 1 to estimate this area, giving your answer correct to 3 significant figures. [1]
- (iii) State, with a reason, which of these estimates is closer to the true area. [1]

- 6 In Fig. 6, triangle ADC is right-angled at C, with $CD = h$. The point B on AC is such that $AB = x$, angle $ADB = \alpha$ and angle $BDC = \beta$.

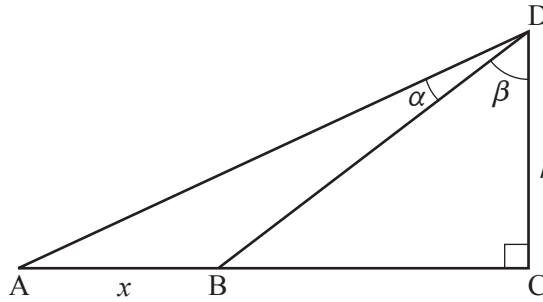


Fig. 6

- (i) Find BC and AC in terms of h , α and β .

Hence show that $x = \frac{h \tan \alpha \sec^2 \beta}{1 - \tan \alpha \tan \beta}$. [5]

- (ii) Given that $x = h$ and $\beta = 30^\circ$, find α , giving your answer correct to 1 decimal place. [3]

Section B (36 marks)

- 7 Three points A, B and C have coordinates A (2, 1, 1), B (1, -3, -1) and C (-4, -1, 0).

- (i) Find the lengths AB and AC, and use a scalar product to calculate the angle BAC.

Hence find the area of triangle ABC. [7]

The lines with vector equations

$$\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k}), \quad \mathbf{r} = \mathbf{i} - 3\mathbf{j} - \mathbf{k} + \mu(-\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = -4\mathbf{i} - \mathbf{j} + \nu(4\mathbf{i} + \mathbf{j} + 2\mathbf{k})$$

pass through the points A, B and C respectively.

- (ii) Show that these three lines meet at a point D. [6]

You are given that the plane ABC has equation $\mathbf{r} \cdot (\mathbf{j} - 2\mathbf{k}) = -1$. The normal through D to the plane ABC meets the plane at E.

- (iii) Find the coordinates of E. [3]

The volume of a tetrahedron is $\frac{1}{3} \times \text{area of base} \times \text{height}$.

- (iv) Find the volume of the tetrahedron ABCD. [3]

- 8 The speed $v \text{ m s}^{-1}$ of an object at time t seconds is modelled by the differential equation

$$\frac{dv}{dt} = -kv(4 + v^2),$$

where k is a positive constant. Initially, $v = 4$.

(i) Find constants A , B and C such that $\frac{1}{v(4 + v^2)} = \frac{A}{v} + \frac{Bv + C}{4 + v^2}$. [5]

(ii) Hence show by integration that $v = \frac{4}{\sqrt{5e^{8kt} - 4}}$. [9]

(iii) After 1 second the speed of the object is 2 m s^{-1} . Find the value of k . [3]

END OF QUESTION PAPER

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Oxford Cambridge and RSA

Friday 22 June 2018 – Morning

A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

OCR supplied materials:

- Question Paper 4754/01A (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



Candidate forename		Candidate surname	
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Centre number						Candidate number				
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Section A (36 marks)

1	

5 (ii)	

5 (iii)	

Section B (36 marks)

7(i)	

7(iii)	
7(iv)	

8 (ii)	(continued)

8 (iii)	



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GCE

Mathematics (MEI)

Unit **4754A**: Applications of Advanced Mathematics: Paper A

Advanced GCE

Mark Scheme for June 2018

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

- a Annotations should be used whenever appropriate during your marking.
The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.
For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.
Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.
- c The following types of marks are available.
- M**
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
- A**
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.
- B**
Mark for a correct result or statement independent of Method marks.
- E**
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.
- Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.
- Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g Rules for replaced work
- If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.
- If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.
- NB Follow these maths-specific instructions rather than those in the assessor handbook.
- h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.
- Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

Question	Answer	Marks	Guidance
1	$\sin \theta - 2.4 \cos \theta = R(\sin \theta \cos \alpha - \cos \theta \sin \alpha)$ $\Rightarrow R \cos \alpha = 1, R \sin \alpha = 2.4$ $R^2 = 1^2 + 2.4^2 = 6.76 \Rightarrow R = 2.6$ $\tan \alpha = 2.4$ $\Rightarrow \alpha = 1.176$ <p>Max value is $1 - (-2.6) = 3.6$</p>	<p>M1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>[5]</p>	<p>Using θ for α can score 4/5 (so loses the A mark) but allow recovery</p> <p>Correct pairs – condone sign errors only</p> <p>Allow $\sqrt{6.76}$ but not $\pm\sqrt{6.76}$ or ± 2.6 unless positive value chosen</p> <p>For M1 follow through their pairs (condone sign errors but division must be the correct way round)</p> <p>A1 for 1.18 or better, with no errors seen in either their method for α or for their pair of equations – A0 if in degrees</p> <p>SC: If candidates state that $\cos \alpha = 1, \sin \alpha = 2.4 \Rightarrow \tan \alpha = 2.4$ this could score M0 B1 M1 A0 B1 (so max 3/5)</p> <p>Note that candidates who state the correct values of R and α with no (wrong) working seen can score full marks</p>
2	$y = \ln x \Rightarrow x = e^y$ $V = (\pi) \int_0^1 x^2 dy = (\pi) \int_0^1 e^{2y} dy$ $= \frac{1}{2} e^{2y}$ $= \left[\frac{1}{2} \pi e^{2y} \right]_0^1 = \frac{1}{2} \pi (e^2 - 1)$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[4]</p>	<p>Seen or implied</p> <p>Condone lack of π and limits for this mark (and dy throughout) – this mark can be awarded if left unsimplified e.g. $\int (e^y)^2 dy$ or even for implying that the required integral is $\int e^{y^2} dy$ - M0 if rotation is about the x-axis</p> <p>Condone + c</p> <p>Exact (oe) – mark final answer (so no isw if correct answer is e.g. halved) but if exact answer seen and is then followed by 10.035... then this mark can be awarded</p>

Question	Answer	Marks	Guidance
3	$\left(\frac{1}{(2-x)^3}\right)k\left(1-\frac{1}{2}x\right)^{-3}$ $= k\left[1+(-3)\left(-\frac{1}{2}x\right)+\frac{(-3)(-4)}{2!}\left(-\frac{1}{2}x\right)^2+\dots\right]$ $= k\left[1+\frac{3}{2}x+\frac{3}{2}x^2+\dots\right]$ $= \frac{1}{8}+\frac{3}{16}x+\frac{3}{16}x^2+\dots$ $\frac{1+2x}{(2-x)^3} = (1+2x)\left[\frac{1}{8}+\frac{3}{16}x+\frac{3}{16}x^2+\dots\right]$ $= \frac{1}{8}+\frac{3}{16}x+\frac{3}{16}x^2+\frac{2}{8}x+\frac{6}{16}x^2+\dots$ $= \frac{1}{8}+\frac{7}{16}x+\frac{9}{16}x^2(+\dots) \text{ or } \frac{1}{8}\left(1+\frac{7}{2}x+\frac{9}{2}x^2(+\dots)\right)$ <p>valid for $x < 2$</p>	<p>B1</p> <p>M1*</p> <p>A1ft</p> <p>A1</p> <p>M1dep*</p> <p>A1</p> <p>B1</p> <p>[7]</p>	<p>Their $k \neq 0$ (for reference the correct k is $\frac{1}{8}$)</p> <p>All three correct unsimplified binomial coefficients (not left as nCr) i.e. 1, -3 and $\frac{(-3)(-4)}{2}$ (allow 2!). Or correct simplified coefficients seen</p> <p>A1ft - correct simplified three-term expression with their k e.g. $k\left(1+\frac{3}{2}x+\frac{3}{2}x^2+\dots\right)$ or for $1+\frac{3}{2}x+\frac{3}{2}x^2+\dots$ seen</p> <p>Correct expansion of $\frac{1}{(2-x)^3}$ - allow with a factor of $\frac{1}{8}$</p> <p>Multiplying out correctly (so multiplying $(1+2x)$ by all three terms to obtain all relevant terms) for their three term expansion of $\frac{1}{(2-x)^3}$</p> <p>www cao oe – ignore any higher order terms stated – do not isw after correct expansion seen if changed e.g. multiplying by 16</p> <p>or $-2 < x < 2$, allow $-2 < x < 2$ but not say, $x < 2$</p>

Question		Answer	Marks	Guidance
4	(i)	$\frac{dx}{d\theta} = 2\cos 2\theta, \quad \frac{dy}{d\theta} = 2\sin 2\theta - 2\sin \theta$ $\frac{dy}{dx} = \frac{2\sin 2\theta - 2\sin \theta}{2\cos 2\theta}$	B1 B1 B1ft [3]	B1 for each Their $\frac{dy}{d\theta}$ divided by their $\frac{dx}{d\theta}$ - dependent on one previous B mark - isw after correct answer seen Note equivalent correct answers e.g. $\frac{\sin 2\theta - \sin \theta}{\cos 2\theta}$, $\tan 2\theta - \sin \theta \sec 2\theta$, etc.
4	(ii)	$2\sin 2\theta - 2\sin \theta = 0 \Rightarrow 4\sin \theta \cos \theta - 2\sin \theta = 0$ $\sin \theta (2\cos \theta - 1) = 0 \Rightarrow \theta = \pi/3 \text{ or } 60$ $x = \sin 2\pi/3 = \sqrt{3}/2$ $y = 1 + 2\cos \pi/3 - \cos 2\pi/3 = 2\frac{1}{2}$	M1 A1 A1 A1 [4]	Setting their numerator or $\frac{dy}{d\theta}$ or derivative equal to zero and use of correct double-angle formula(e) to remove all double angles – note that use of correct double-angle formula(e) may be seen in part (i) Condone additional solutions from either $\sin \theta = 0$ or $2\cos \theta - 1 = 0$ - ignore any mention of, or lack of consideration of $\sin \theta = 0$, condone $\theta = 60$ Must be exact and simplified – condone if given as a decimal provided exact value seen – must come for correct working Must be exact and simplified – must come from correct working If more than one answer given then award M1A1A1A0 max. SC1: B2 for $\sin \theta = \sin 2\theta \Rightarrow \theta = \pi/3$ and then B1 for x and B1 for y SC2: After M1 A1 award B1 for $y = \sqrt{3}/2$ and $x = 2\frac{1}{2}$ (unless recovered) provided no additional solutions

Question		Answer	Marks	Guidance
5	(i)	0.5×0.25 $[0 + 1 + 2(0.7603 + 0.9354 + 0.9922)]$ or $\left[0 + 1 + 2 \left(\frac{\sqrt{37}}{8} + \frac{\sqrt{14}}{4} + \frac{3\sqrt{7}}{8} \right) \right]$ 0.797	B1 M1 A1 [3]	For using 0.125 oe The M mark requires the correct [...] bracket structure. It needs the first bracket to contain the first y value (if present must be zero but condone its absence) plus the last y value and the second bracket to be multiplied by 2 and to be the summation of the remaining y values with no additional values. Allow an error in one value or the omission of one value from the second bracket. M0 if using <u>all</u> x values. Candidates must be working to at least 2 decimal places Must be given correct to 3 significant figures only (for reference 'correct' answer is 0.79697910...) Correct answer with no working is 0/3
5	(ii)	$A \approx \frac{1}{4} \times \pi \times 1^2 = 0.785$	B1 [1]	Must be given correct to 3 significant figures only
5	(iii)	The <u>trapezium rule</u> estimate is <u>closer</u> , as it is an <u>under-estimate</u> , but is <u>greater</u> than the <u>quarter circle estimate</u>	B1 [1]	Must contain the fact that the trapezium rule which is an under-estimate of the area is closer to the true area as it is greater in value than the quarter-circle area This mark is dependent on both correct values to at least 3 sf seen in parts (i) and (ii)

Question	Answer	Marks	Guidance
6 (i)	$(BC =) h \tan \beta$ $(AC =) h \tan(\alpha + \beta)$ <p>So $x = h[\tan(\alpha + \beta) - \tan \beta] = h\left(\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} - \tan \beta\right)$</p> $= h\left(\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} - \frac{\tan \beta - \tan \alpha \tan^2 \beta}{1 - \tan \alpha \tan \beta}\right) \text{ or}$ $h\left(\frac{\tan \alpha + \tan \beta - \tan \beta(1 - \tan \alpha \tan \beta)}{1 - \tan \alpha \tan \beta}\right)$ $= h\frac{\tan \alpha + \tan \alpha \tan^2 \beta}{1 - \tan \alpha \tan \beta} = \frac{h \tan \alpha(1 + \tan^2 \beta)}{1 - \tan \alpha \tan \beta}$ $= \frac{h \tan \alpha \sec^2 \beta}{1 - \tan \alpha \tan \beta}$	<p>B1</p> <p>B1</p> <p>M1*</p> <p>M1dep*</p> <p>A1</p> <p>[5]</p>	<p>BC and AC must be explicit expressions and not just seen as part of an equation/formula</p> <p>Correct expansion of compound-angle formula for $\tan(\alpha + \beta)$ seen</p> <p>Either combining both fractions correctly or re-writing the second term with a denominator of $1 - \tan \alpha \tan \beta$</p> <p>NB AG – www dependent on all previous marks – the identity $1 + \tan^2 \beta = \sec^2 \beta$ need not be stated explicitly but the numerator must be factorised before the given result or by replacing $\tan^2 \beta$ with $\sec^2 \beta - 1$. Any incorrect working (even if recovered) loses this accuracy mark (as answer given)</p>
6 (ii)	$h \left(\frac{\frac{4}{3} \tan \alpha}{1 - \frac{\sqrt{3}}{3} \tan \alpha} \right) = h$ $\frac{4}{3} \tan \alpha + \frac{\sqrt{3}}{3} \tan \alpha = 1$ $\alpha = 27.6^\circ$	<p>M1*</p> <p>M1dep*</p> <p>A1</p> <p>[3]</p>	<p>Correctly substituting of $x = h$ (or $h = x$) and $\beta = 30$ – allow $h \left(\frac{1.33 \tan \alpha}{1 - 0.577 \tan \alpha} \right) = h$ or better. If 30 substituted correctly allow one error in the evaluation of $\tan 30$ or $\sec^2 30$</p> <p>Cancellation of h (or x) from both sides, cross-multiplying and rearranging to get both terms in $\tan \alpha$ on the same side – allow 1.33 and 0.577</p> <p>27.6 or better (degree symbol not required) – A0 for answer in radians - for reference 27.626340...</p> <p>Alternative method:</p> <p>B1 for $BC = h \tan 30$</p> <p>M1 for $\tan(\alpha + 30) = \frac{h + BC}{h}$ and their BC must contain h and $\tan 30$</p> <p>A1 for 27.6</p>

Question	Answer	Marks	Guidance
7 (i)	$AB = \sqrt{(2-1)^2 + (1-(-3))^2 + (1-(-1))^2} = \sqrt{21}$ $AC = \sqrt{(2-(-4))^2 + (1-(-1))^2 + (1-0)^2} = \sqrt{41}$ $\vec{AB} = \begin{pmatrix} -1 \\ -4 \\ -2 \end{pmatrix}, \vec{AC} = \begin{pmatrix} -6 \\ -2 \\ -1 \end{pmatrix} \Rightarrow \vec{AB} \cdot \vec{AC} = 6 + 8 + 2 (= 16)$ $\cos \theta = \frac{16}{\sqrt{21} \cdot \sqrt{41}}$ <p>Angle BAC is 57.0 or 0.994</p> $\text{Area} = \frac{1}{2} \times \sqrt{21} \times \sqrt{41} \sin 56.96$ $= 12.3$	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1*</p> <p>A1</p> <p>M1dep*</p> <p>A1</p> <p>[7]</p>	<p>Allow 4.58 or better (4.5825756...) – must be explicitly stated and not just as part of a scalar product</p> <p>Allow 6.4(0) or better (6.4031242...) – must be explicitly stated and not just as part of a scalar product</p> <p>Correct product - allow full marks for using direction vectors BA and CA</p> <p>Must be using their vectors AB, BA, AC or CA only - correct use of scalar product by combining their values correctly including cosine, use of cosine rule is M0</p> <p>www - allow 57 or better (for reference 56.956299...) – do not allow this mark for an answer which rounds to 56.9 – accept 0.994 or better (radians)</p> <p>Correct use of area formula $0.5ab \sin C$ with their values</p> <p>www – for reference 12.298373... allow from 12.30444... (which comes from using 57)</p> <p>SC: If first method mark not awarded for use of cosine rule then award B1 for correct area</p> <p>Please look carefully for candidates misreading (MR) the coordinates of A, B and/or C</p>

Question	Answer	Marks	Guidance
7 (ii)	$2+2\lambda=1-\mu \quad 2+2\lambda=-4+4\nu \quad 1-\mu=-4+4\nu$ $1+\lambda=-3+3\mu \text{ or } 1+\lambda=-1+\nu \text{ or } -3+3\mu=-1+\nu$ $1-\lambda=-1+3\mu \quad 1-\lambda=2\nu \quad -1+3\mu=2\nu$ <p>For first set of equations a value of λ or μ obtained from valid method (or equivalent parameters for second or third set) Two of $\lambda = -1, \mu = 1, \nu = 1$</p> <p>e.g. $1 - (-1) = -1 + 3(1)$ oe</p> $\text{e.g. } \mathbf{r} = \begin{pmatrix} -4 \\ -1 \\ 0 \end{pmatrix} + \nu \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \Rightarrow \nu = 1$	<p>B1</p> <p>M1</p> <p>A2</p> <p>A1</p> <p>A1</p> <p>[6]</p>	<p>3 correct equations from the same set (i.e. in the same 2 unknowns) – note that the third equation may appear later, or with values already substituted. Note that this mark could be awarded when (some) candidates derive/show the coordinate of D by showing that two lines give the same coordinate when the values of the parameters are substituted into all three equations</p> <p>A1 for one correct value</p> <p>Correct substitution into third equation (ie the one not used to find the two previous values) to verify that two of the lines meet (at D). In essence this mark is for verifying at some stage that two of the lines meet at a point by showing that all three equations with the correct two parameters are consistent</p> <p>Either using D(0,0,2) to find the third value of either λ, μ or ν for <u>all</u> three relevant equations (not just one) or solving a different set of equations and obtaining a different pair of $\lambda = -1, \mu = 1, \nu = 1$ and confirming that they meet at the same point. In essence this mark is for confirming that all three lines meet at the same point and therefore this mark is dependent on all previous marks Note that substituting all three correct values of the scalar parameters into all 3 sets of equations and obtaining the correct D (or showing that they are all equal) is acceptable and scores full marks</p> <p>Note: no MRs in this part</p>

Question		Answer	Marks	Guidance
7	(iii)	$\text{(Normal is } \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + k \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$ $\begin{pmatrix} 0 \\ k \\ 2-2k \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} = -1 \Rightarrow k - 2(2 - 2k) = -1$ $\Rightarrow 5k = 3, k = 0.6$ $\Rightarrow E \text{ is } (0, 0.6, 0.8)$	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>Do not need to see $\mathbf{r} = \dots$ allow any scalar parameter for k</p> <p>Substitutes their equation of the normal (allow any position vector including zero/absent) into the plane and evaluate dot product – the direction vector of the normal though D to the plane must be correct and must contain a scalar parameter (allow any letter used) or $y - 2z = -1 \Rightarrow k - 2(2 - 2k) = -1$</p> <p>Condone stated as a position vector – mark final answer so no isw if multiplied by say 5</p>
7	(iv)	$DE = \sqrt{(0.6^2 + 1.2^2)} (= 1.3416\dots)$ <p>Volume of tetrahedron = $(1.3416 \times 12.3) / 3$</p> <p>5.5</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>For reference D(0, 0, 2) and E(0, 0.6, 0.8) – www</p> <p>Correct use of formula with their area from part (i)</p> <p>Accept 5.5 or answers which round to 5.50</p>
8	(i)	$\frac{1}{v(4+v^2)} = \frac{A}{v} + \frac{Bv+C}{4+v^2}$ $1 = A(4+v^2) + (Bv+C)v$ $v = 0 \Rightarrow 1 = 4A, A = \frac{1}{4}$ <p>coefficient of v^2: $0 = A + B \Rightarrow B = -\frac{1}{4}$</p> <p>coefficient of v: $0 = Cv \Rightarrow C = 0$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[5]</p>	<p>Seen or implied (allow recovery)</p> <p>Cover up, substitution or equating coefficients – must be a complete method for finding one of A, B or C</p> <p>www - these accuracy marks for the values of A, B or C must be checked carefully as it is possible to get these values correct with a variety of incorrect working</p> <p>www</p> <p>www</p> <p>Note that correct values for A, B and C from incorrect working will lose the subsequent A marks in (ii)</p>

Question	Answer	Marks	Guidance
8 (ii)	$\int \frac{dv}{v(4+v^2)} = -\int k dt$ <p>For reference $\int \left[\frac{1}{4v} - \frac{v}{4(4+v^2)} \right] dv = -kt + c$</p> $\int \frac{\lambda v}{(4+v^2)} dv = \frac{\lambda}{2} \ln(4+v^2), \int \frac{\mu}{v} dv = \mu \ln v$ $\frac{1}{4} \ln v - \frac{1}{8} \ln(4+v^2) = -kt (+c)$ <p>when $t = 0, v = 4 \Rightarrow c = \frac{1}{4} \ln 4 - \frac{1}{8} \ln 20$</p> $\Rightarrow \frac{1}{8} \ln \frac{v^2}{4+v^2} = -kt - \frac{1}{8} \ln \frac{5}{4} \Rightarrow \ln \frac{v^2}{4+v^2} + \ln \frac{5}{4} = -8kt$ $\Rightarrow \ln \left(\frac{5v^2}{4(4+v^2)} \right) = -8kt$ $\Rightarrow \frac{5v^2}{4(4+v^2)} = e^{-8kt} \text{ or } \frac{v^2}{4+v^2} = \frac{4}{5} e^{-8kt}$ $\frac{4+v^2}{v^2} = \frac{5}{4} e^{8kt} = \frac{4}{v^2} + 1$ $\Rightarrow \frac{4}{v^2} = \frac{5}{4} e^{8kt} - 1 \Rightarrow \frac{v^2}{4} = \frac{4}{5e^{8kt} - 4}$ $\Rightarrow v^2 = \frac{16}{5e^{8kt} - 4}, v = \frac{4}{\sqrt{5e^{8kt} - 4}}^*$	<p>M1*</p> <p>B1ft B1ft A1</p> <p>M1dep*</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[9]</p>	<p>Separating variables - condone sign slips and issues with placement/lack of of k but M0 for $\int v(4+v^2) dv = \dots$ or equivalent algebraic error in separating variables unless recovered. If no subsequent work integral signs needed, but allow omission of dv and/or dt but must be correctly placed if present</p> <p>Any non-zero λ, μ following through from their values in part (i) oe</p> <p>oe (e.g. $\frac{1}{4} \ln 4v$) - do not condone invisible brackets e.g. $\ln 4 + v^2$ unless recovered later – condone absence of c (no follow through on this mark)</p> <p>Substituting $v = 4, t = 0$ correctly into each of their terms in an attempt to find their c (must get $c = \dots$)</p> <p>Correctly combines all their log terms (must include two log terms from integration and at least one from finding c) – dependent on all previous M marks. This mark can also be awarded for removing logs correctly from all terms. M0 if $C \neq 0$ from part (i)</p> <p>Correct equation (without logs)</p> <p>Correct method for making v or v^2 the subject– dependent on all previous M marks</p> <p>AG – all previous marks must have been awarded</p> <p>PLEASE ENSURE THAT THE SECOND PAGE OF THIS PART IS CHECKED AND IF NOT USED IS ANNOTATED WITH BP</p>

Question		Answer	Marks	Guidance
8	(iii)	$2 = \frac{4}{\sqrt{5e^{8k} - 4}}$ $\sqrt{5e^{8k} - 4} = 2 \Rightarrow 5e^{8k} = 8 \Rightarrow e^{8k} = \frac{8}{5}$ $\Rightarrow k = \frac{1}{8} \ln \frac{8}{5} \text{ or } 0.059$	B1 M1 A1 [3]	or correct equivalent e.g. $\ln \frac{4}{4+4} + \ln \frac{5}{4} = -8k$ - substitution of values may be seen later Correct method of getting e^{8k} equal to a non-zero constant (allow sign slips only or for $e^{8k} = \frac{5}{8}$ from $5e^{8k} = 8$) or making k the subject Or exact equivalent e.g. $= -\frac{1}{8} \ln \frac{5}{8}$ - accept 0.059 or better

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AS/A LEVEL GCE

Examiners' report

MATHEMATICS (MEI)

3895-3898, 7895-7898

4754/01A Summer 2018 series

Version 1

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Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper 4754/01A series overview

Applications of Advanced Mathematics (core 4) 4754 is the fourth mandatory component of 7895 A Level Mathematics (MEI). This component is made up of this examination paper and a separate comprehension task. This is the final assessment series for this specification, although there is a resit opportunity in summer 2019.

On the whole, candidates found Paper A this year slightly more demanding than last year although the standard of work in the majority of cases was very high. This paper was accessible to all candidates but there were sufficient questions for the more able candidates to show their skills.

Candidates made similar errors as in previous years and these included:

- Sign and basic algebraic errors (Questions 3, 4, 6(i), 6(ii), 8(ii) and 8(iii))
- Failure to include a constant of integration (Question 8(ii))
- Inappropriate accuracy, for example in Question 5 (i) and (ii), candidates either gave insufficient accuracy (answers to 2 significant figures) or they gave too much accuracy (answers to 4 or more significant figures). Candidates are reminded to give answers to 1 decimal place for questions involving trigonometry (Questions 6(ii) and 7(i))
- Failure to give exact answers when required (Question 2 and 4(ii))
- Failure to give sufficient detail when verifying given results (Questions 6(i) and 8(ii))

Quite a number of candidates did not attempt some parts but there did not appear to be a shortage of time.

Centres are again reminded that as Papers A and B are marked separately any supplementary sheets used must be attached to the appropriate paper. Furthermore, centres are requested that Papers A and B are not attached to each other and are sent separately for marking.

Section A overview

Section A focuses upon routine calculations and structured problem solving questions. This section is worth 36 marks. Candidates made a good attempt at all the questions in Section A, with very few parts of questions with no response provided.

Question 1

- 1 Express $\sin \theta - 2.4 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence write down the maximum value of the function $f(\theta) = 1 - \sin \theta + 2.4 \cos \theta$, where $0 \leq \theta \leq 2\pi$. [5]

The majority of candidates correctly calculated the values of R and α although some lost the first method mark by not including R in the expanded trigonometric statements $R \cos \alpha = 1$ and $R \sin \alpha = 2.4$. Some candidates did not give α in radians and a small minority stated R as 6.76 rather than the correct 2.6. While candidates found the first four marks in this part relatively straightforward many could not write down the maximum value of $f(\theta)$ even though the question gave the hint of 'hence'; many either gave the maximum as 2.6 (the value of R) or as 3.4 rather than realising that $f(\theta) = 1 - 2.6 \sin(\theta - 1.17\dots) \Rightarrow f_{\max} = 1 - 2.6(-1) = 3.6$.

Question 2

- 2 The finite region bounded by the curve $y = \ln x$, the x -axis, the y -axis and the line $y = 1$ is rotated through 360° about the y -axis. Find the exact volume of the solid of revolution generated. [4]

In this question the vast majority of candidates considered both the correct integral (with correct limits) for the volume of revolution generated by rotating the given curve about the y -axis and went on to integrate correctly. A number of candidates, however, misread the question and instead tried to calculate the volume of revolution generated by rotating the curve about the x -axis. While nearly all candidates who correctly rotated about the y -axis stated that $V = \pi \int_0^1 (e^y)^2 dy$ a number then wrote

$$\int_0^1 e^{y^2} dy = \left[\frac{1}{3} e^{y^3} \right]_0^1 \text{ or, for those that did have the correct integral, some stated that } \int_0^1 e^{2y} dy = \left[2e^{2y} \right]_0^1.$$

Of those that did integrate correctly a number forgot the π in their final answer or did not give an exact answer. Finally, a number of candidates, who had the correct answer of $\frac{1}{2} \pi (e^2 - 1)$ then, for some inexplicable reason, went on to halve (or even double) their answer.

Question 3

- 3 Find the first three terms of the binomial expansion of $\frac{1+2x}{(2-x)^3}$ in ascending powers of x .

State the set of values of x for which the expansion is valid.

[7]

The vast majority of candidates correctly re-wrote the expression $\frac{1+2x}{(2-x)^3}$ as $(1+2x)(2-x)^{-3}$ and then went on to re-write $(2-x)^{-3}$ as $\frac{1}{8}\left(1-\frac{1}{2}x\right)^{-3}$. However, some candidates either expanded $(2-x)^3$ in the denominator of the original expression, stated that $(2-x)^{-3} = 1+(-3)(-x)+\dots$ or tried (mostly without success) to express the original expression using partial fractions. While some candidates struggled with the factor of $\frac{1}{8}$, most correctly expanded $\left(1-\frac{1}{2}x\right)^{-3}$ (although the third term for some contained a binomial coefficient of $\frac{(-3)(-2)}{2!}$). Most candidates, after expanding $(2-x)^{-3}$, went on to successfully multiply their expansion for $(2-x)^{-3}$ with $(1+2x)$ and therefore obtained the first three terms of the required binomial expansion.

Finally, a number of candidates either did not state the set of values of x for which the expansion was valid or gave an answer of $|x| < \frac{1}{2}$ either on its own or together with the correct answer of $|x| < 2$.

Question 4(i)

- 4 A curve has parametric equations $x = \sin 2\theta$, $y = 1 + 2 \cos \theta - \cos 2\theta$, where $0 < \theta < \pi$.

(i) Find $\frac{dy}{dx}$ in terms of θ .

[3]

Apart from the standard errors in trigonometric differentiation most candidates differentiated both terms correctly and then divided $\frac{dy}{d\theta}$ by $\frac{dx}{d\theta}$ to obtain the required $\frac{dy}{dx}$ in terms of θ . Some, however, began this part by attempting to expand the double-angles before differentiation (and many of these attempts were not successful). Finally, a number of candidates, after having stated the correct derivative, spent time unnecessarily 'simplifying' their expression and, in some cases, introducing unnecessary errors.

Question 4(ii)

(ii) Find the exact coordinates of the point on the curve where the gradient is zero.

[4]

While it was pleasing to note that nearly all candidates recognised that only the numerator of their algebraic fraction from part (i) needed to be put equal to zero many struggled with solving the corresponding trigonometric equation with many writing $\sin 2\theta = \sin \theta \Rightarrow \theta = 0$ and hence gaining no marks in this part. Of those that applied the correct double-angle formula many did obtain the correct $\theta = \frac{\pi}{3}$ and hence the correct (exact) values of x and y . While it was pleasing to see many candidates considering the equation $\sin \theta (2\cos \theta - 1) = 0$ (rather than immediately dividing by $\sin \theta$) a number gave a solution of $(0, 2)$ which came from a value of theta (that is $\theta = 0$) that was unfortunately not in the required range.

Question 5(i)

5 Fig. 5 shows the curve with equation $y = \sqrt{1+x^3}$.

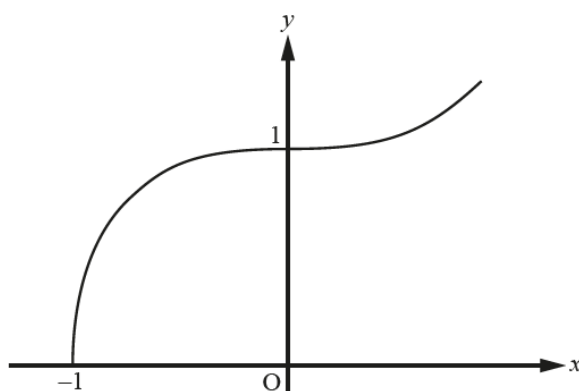


Fig. 5

(i) Use the trapezium rule with 4 strips to estimate the finite area enclosed by the curve and the x - and y -axes, giving your answer correct to 3 significant figures. [3]

Part (i) was answered extremely well with the vast majority of candidates giving the correct answer of 0.797. When errors occurred, it was usually due to an incorrect value for the width of the strips or with the omission of a value. It was very rare for candidates to use the x values or to not give the answer to the required 3 significant figures.

Question 5(ii)

(ii) Use a quarter circle of radius 1 to estimate this area, giving your answer correct to 3 significant figures. [1]

Nearly all candidates achieved the correct answer of 0.785 although some did not give the answer to the required 3 significant figures, instead giving it to at least 4 significant figures or as a multiple of π . There were a small proportion of candidates that used incorrect formulae for the area of a circle.

Question 5(iii)

(iii) State, with a reason, which of these estimates is closer to the true area.

[1]

While approximately half of the candidates correctly stated that the trapezium rule was the closer estimate of the two this therefore meant that approximately half of the candidates stated that the quarter circle was closer to the true area. Of those that did state that the trapezium rule was closer many did not give sufficient detail for why it provided a better estimate for the true area. Examiners needed to see mention of the fact that although the trapezium rule gives an underestimate of the area it is still greater in value than the quarter circle estimate.

Question 6(i)

6 In Fig. 6, triangle ADC is right-angled at C, with $CD = h$. The point B on AC is such that $AB = x$, angle $ADB = \alpha$ and angle $BDC = \beta$.

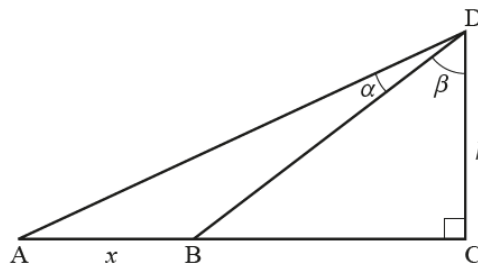


Fig. 6

(i) Find BC and AC in terms of h , α and β .

Hence show that $x = \frac{h \tan \alpha \sec^2 \beta}{1 - \tan \alpha \tan \beta}$.

[5]

While many candidates did state correct expressions for BC and AC in terms of h, α and β , a number gave an incorrect answer of $BC = \frac{h}{\tan \beta}$ (together with a similar incorrect expression for AC), or did not give explicit expressions for these two lengths. It was also relatively common to see an expression for AC given in terms of x . The majority of candidates stated that $x = h(\tan(\alpha + \beta) - \tan \beta)$ and most correctly expanded $\tan(\alpha + \beta)$. While most went on to correctly combine both fractions only the most accurate of candidates obtained full marks for correctly obtaining (without errors) the given answer for x .

Question 6(ii)

(ii) Given that $x = h$ and $\beta = 30^\circ$, find α , giving your answer correct to 1 decimal place.

[3]

In questions such as this, candidates are strongly advised to immediately substitute the given values before attempting to re-arrange as this makes the resulting re-arrangement and simplification a lot easier to complete. Also, many candidates did not see the natural link between the answer given in part (i) and the demand in part (ii). While the majority of candidates correctly stated the value of α correct to 1 decimal place it was slightly worrying the number of candidates who either explicitly stated (or implied) that $\sec^2 \beta = \frac{1}{\tan^2 \beta}$.

Section B overview

Section B has two longer questions with a more problem solving focus. These questions proved challenging for many candidates, but it was pleasing to see candidates making a good attempt at the majority of parts ensuring partial credit, even if the response did not progress to a complete solution.

Question 7(i)

7 Three points A, B and C have coordinates A (2, 1, 1), B (1, -3, -1) and C (-4, -1, 0).

(i) Find the lengths AB and AC, and use a scalar product to calculate the angle BAC.

Hence find the area of triangle ABC.

[7]

It was pleasing to note that the majority of candidates scored extremely well on this part with many scoring full marks, even though this was a multi-step solution. A number of candidates didn't state explicitly the lengths of AB and AC as requested and there were a significant number of candidates made transpose errors copying the coordinates from the question. The question specifically required candidates to use the scalar product, but a significant number of responses used the cosine rule to find this angle. The most common method error in this part was for those candidates who used incorrect direction vectors or simply used two of the three points as direction vectors in their scalar product. While most used $\frac{1}{2}ab\sin C$ correctly to find the area of triangle ABC a number either used an incorrect form of this equation (for example, some used cosine instead of sine and some forgot the half) while some tried (mostly unsuccessfully) to use the formula half base times height.

Question 7(ii)

The lines with vector equations

$$\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k}), \quad \mathbf{r} = \mathbf{i} - 3\mathbf{j} - \mathbf{k} + \mu(-\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = -4\mathbf{i} - \mathbf{j} + \nu(4\mathbf{i} + \mathbf{j} + 2\mathbf{k})$$

pass through the points A, B and C respectively.

(ii) Show that these three lines meet at a point D.

[6]

While some candidates gave textbook answers to this part and showed conclusively that all three lines would meet at the point D most did not show sufficient working. In fact, examiners commented that in many scripts the candidates' work was unclear making it extremely difficult to read and understand at times. The most common way of showing this required result was to first show that two of the lines did indeed meet at a point. This should have been done by first showing that there exists unique scalar parameters such that all three corresponding equations for those two lines are consistent. While many candidates started this method correctly by taking two of the equations and solving to find the required parameters they never showed consistency in the third equation, therefore never showing that two of the lines did indeed meet at a point. Of those that did show conclusively that two of the lines did meet at a point many did go on to either solve for the third parameter and then show that this value gave the same point as before or they proceeded to set the third line equal to D and showed that all three equations in the final parameter were consistent. The most common error was from those candidates who thought that simply finding D was equivalent to showing that all three lines meet at a point (and in many cases this only gained three of the six marks available).

Question 7(iii)

You are given that the plane ABC has equation $\mathbf{r} \cdot (\mathbf{j} - 2\mathbf{k}) = -1$. The normal through D to the plane ABC meets the plane at E.

(iii) Find the coordinates of E. [3]

Part (iii) and part (iv) were the only questions where a significant number of candidates did not provide any response. Of those candidates that did attempt part (iii); very few correctly substituted the line through D normal to the plane into the equation of the plane. Therefore, the majority did not solve a linear equation to find the scalar parameter which then needed to be substituted back into the line normal to the plane to obtain the coordinates of E as (0, 0.6, and 0.8).

Question 7(iv)

The volume of a tetrahedron is $\frac{1}{3} \times \text{area of base} \times \text{height}$.

(iv) Find the volume of the tetrahedron ABCD. [3]

Although a significant number of candidates made no attempt on this part, a good number of candidates were able to score at least the method mark for correctly calculating $\frac{1}{3}(\text{their DE})(\text{their area from part (i)})$.

Question 8(i)

8 The speed $v \text{ m s}^{-1}$ of an object at time t seconds is modelled by the differential equation

$$\frac{dv}{dt} = -kv(4 + v^2),$$

where k is a positive constant. Initially, $v = 4$.

(i) Find constants A , B and C such that $\frac{1}{v(4 + v^2)} = \frac{A}{v} + \frac{Bv + C}{4 + v^2}$. [5]

This part was answered extremely well with many candidates scoring full marks. However, a number of candidates obtained the correct values of A , B and C from incorrect working which then had a knock-on effect with regards to the accuracy marks in part (ii).

Question 8(ii)

(ii) Hence show by integration that $v = \frac{4}{\sqrt{5e^{8kt} - 4}}$.

[9]

This part was the probably the most demanding part of the paper and many candidates made very little progress after correctly separating the variables and writing $\int \frac{dv}{v(4+v^2)} = -\int k dt$. Of those that did use

part (i) many incorrectly wrote $\frac{1}{4} \int \frac{1}{v} - \frac{1}{4+v^2} dv = -kt + c$ or could not deal with the required integration.

Of those that obtained a correctly integrated expression, for example, $\frac{1}{4} \ln v - \frac{1}{8} \ln(4+v^2) = -kt + c$ some either forgot the constant of integration or assumed it was zero. Many candidates who did work out their constant correctly and had a correct equation, for example, $\frac{1}{4} \ln v - \frac{1}{8} \ln(4+v^2) = -kt + \frac{1}{4} \ln 4 - \frac{1}{8} \ln 20$

then struggled with the algebra required to either combine the log terms or to remove the logs correctly from all terms. Very few candidates obtained a correct equation without logs, for example,

$\frac{5v^2}{4(4+v^2)} = e^{-8kt}$ and of those that did many then struggled to make v^2 and then v the subject.

Question 8(iii)

(iii) After 1 second the speed of the object is 2 ms^{-1} . Find the value of k .

[3]

It was pleasing to note that many candidates who had struggled with part (ii) realised that they could still access the marks in part (iii). Many correctly substituted $t = 1$ and $v = 2$ into the given answer from part (ii) and most could then re-arrange correctly to make e^{8k} the subject. From this point, most then went on to correctly state k either exactly or correct to at least three decimal places.

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AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

AS & Advanced GCE Mathematics				Max Mark	a	b	c	d	e	u
4721	01	C1 Core mathematics 1 (AS)	Raw	72	61	55	50	45	40	0
			UMS	100	80	70	60	50	40	0
4722	01	C2 Core mathematics 2 (AS)	Raw	72	55	49	43	37	31	0
			UMS	100	80	70	60	50	40	0
4723	01	C3 Core mathematics 3 (A2)	Raw	72	55	48	41	34	28	0
			UMS	100	80	70	60	50	40	0
4724	01	C4 Core mathematics 4 (A2)	Raw	72	54	47	40	34	28	0
			UMS	100	80	70	60	50	40	0
4725	01	FP1 Further pure mathematics 1 (AS)	Raw	72	56	50	45	40	35	0
			UMS	100	80	70	60	50	40	0
4726	01	FP2 Further pure mathematics 2 (A2)	Raw	72	59	53	47	41	35	0
			UMS	100	80	70	60	50	40	0
4727	01	FP3 Further pure mathematics 3 (A2)	Raw	72	47	41	36	31	26	0
			UMS	100	80	70	60	50	40	0
4728	01	M1 Mechanics 1 (AS)	Raw	72	60	51	42	34	26	0
			UMS	100	80	70	60	50	40	0
4729	01	M2 Mechanics 2 (A2)	Raw	72	53	46	39	32	26	0
			UMS	100	80	70	60	50	40	0
4730	01	M3 Mechanics 3 (A2)	Raw	72	50	42	34	27	20	0
			UMS	100	80	70	60	50	40	0
4731	01	M4 Mechanics 4 (A2)	Raw	72	59	53	47	42	37	0
			UMS	100	80	70	60	50	40	0
4732	01	S1 – Probability and statistics 1 (AS)	Raw	72	57	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4733	01	S2 – Probability and statistics 2 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4734	01	S3 – Probability and statistics 3 (A2)	Raw	72	59	50	41	32	24	0
			UMS	100	80	70	60	50	40	0
4735	01	S4 – Probability and statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4736	01	D1 – Decision mathematics 1 (AS)	Raw	72	55	48	42	36	30	0
			UMS	100	80	70	60	50	40	0
4737	01	D2 – Decision mathematics 2 (A2)	Raw	72	58	53	48	44	40	0
			UMS	100	80	70	60	50	40	0

AS & Advanced GCE Mathematics (MEI)			Max Mark	a	b	c	d	e	u	
4751	01	C1 – Introduction to advanced mathematics (AS)	Raw	72	60	55	50	45	40	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – Concepts for advanced mathematics (AS)	Raw	72	53	47	41	36	31	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) Methods for Advanced Mathematics (A2): Written Paper	Raw	72	61	56	51	46	40	0
4753	02	(C3) Methods for Advanced Mathematics (A2): Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) Methods for Advanced Mathematics (A2): Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – Applications of advanced mathematics (A2)	Raw	90	63	56	49	43	37	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – Further concepts for advanced mathematics (AS)	Raw	72	55	51	47	43	40	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – Further methods for advanced mathematics (A2)	Raw	72	48	42	36	31	26	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – Further applications of advanced mathematics (A2)	Raw	72	63	56	49	42	35	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) Differential Equations (A2): Written Paper	Raw	72	61	54	48	42	35	0
4758	02	(DE) Differential Equations (A2): Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) Differential Equations (A2): Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – Mechanics 1 (AS)	Raw	72	51	44	37	31	25	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – Mechanics 2 (A2)	Raw	72	59	53	47	41	35	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – Mechanics 3 (A2)	Raw	72	61	54	48	42	36	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – Mechanics 4 (A2)	Raw	72	59	51	44	37	30	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – Statistics 1 (AS)	Raw	72	59	53	47	42	37	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – Statistics 2 (A2)	Raw	72	54	47	41	35	29	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – Statistics 3 (A2)	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – Decision mathematics 1 (AS)	Raw	72	50	44	38	32	26	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – Decision mathematics 2 (A2)	Raw	72	55	51	47	43	39	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) Numerical Methods (AS): Written Paper	Raw	72	57	52	48	44	39	0
4776	02	(NM) Numerical Methods (AS): Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) Numerical Methods (AS): Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
			UMS	100	80	70	60	50	40	0

AS GCE Statistics (MEI)			Max Mark	a	b	c	d	e	u
G241	01	Statistics 1 MEI	Raw	72	No entry in June 2018				
			UMS	100	80	70	60	50	40
G242	01	Statistics 2 MEI	Raw	72	No entry in June 2018				
			UMS	100	80	70	60	50	40
G243	01	Statistics 3 MEI	Raw	72	No entry in June 2018				
			UMS	100	80	70	60	50	40

AS GCE Quantitative Methods (MEI)			Max Mark	a	b	c	d	e	u	
G244	01	Introduction to Quantitative Methods (Written Paper)	Raw	72	58	50	43	36	28	0
			UMS	100	80	70	60	50	40	0
G244	02	Introduction to Quantitative Methods (Coursework)	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
G245	01	Statistics 1	Raw	72	61	55	49	43	37	0
			UMS	100	80	70	60	50	40	0
G246	01	Decision Mathematics 1	Raw	72	50	44	38	32	26	0
			UMS	100	80	70	60	50	40	0

Level 3 Certificate, Level 3 Extended Project and FSMQ raw mark grade boundaries June 2018 series

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Level 3 Certificate Mathematics - Quantitative Methods (MEI)

					Max Mark	a	b	c	d	e	u
G244	A	01	Introduction to Quantitative Methods with Coursework (Written Paper)	Raw	72	58	50	43	36	28	0
G244	A	02	Introduction to Quantitative Methods with Coursework (Coursework)	Raw	18	14	12	10	8	7	0
				UMS	100	80	70	60	50	40	0
				Overall	90	72	62	53	44	35	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI)

					Max Mark	a	b	c	d	e	u
H866		01	Introduction to quantitative reasoning	Raw	72	56	49	42	35	28	0
H866		02	Critical maths	Raw	60	44	39	34	29	24	0
*To create the overall boundaries, component 02 is weighted to give marks out of 72				Overall	144	109	96	83	70	57	0

Level 3 Certificate Mathematics - Quantitative Problem Solving (MEI)

					Max Mark	a	b	c	d	e	u
H867		01	Introduction to quantitative reasoning	Raw	72	56	49	42	35	28	0
H867		02	Statistical problem solving	Raw	60	40	36	32	28	24	0
*To create the overall boundaries, component 02 is weighted to give marks out of 72				Overall	144	104	92	80	69	57	0

Advanced Free Standing Mathematics Qualification (FSMQ)

					Max Mark	a	b	c	d	e	u
6993		01	Additional Mathematics	Raw	100	56	50	44	38	33	0

Intermediate Free Standing Mathematics Qualification (FSMQ)

					Max Mark	a	b	c	d	e	u
6989		01	Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0

OCR

Oxford Cambridge and RSA

Friday 22 June 2018 – Morning

A2 GCE MATHEMATICS (MEI)

4754/01 Applications of Advanced Mathematics (C4)

INSTRUCTIONS



The examination is in two parts:

Paper A (1 hour 30 minutes)

Paper B (up to 1 hour)

Supervisors are requested to ensure that Paper B **is not issued** until Paper A has been collected in from the candidates.

Centres may, if they wish, grant a supervised break between the two parts of this examination.

Paper B should not be attached to the corresponding Paper A script. For Paper A only the candidates' Printed Answer Books, in the same order as the attendance register, should be sent for marking; the Question Paper should be retained in the centre or recycled. For Paper B only the Question Papers, on which the candidates have written their answers, should be sent for marking; the Insert should be retained in the centre or recycled. Any additional sheets used must be carefully attached to the correct paper.

For Paper B (Comprehension) only.

A standard English dictionary is allowed for the comprehension.

(Dictionaries to be used in the examination must be thoroughly checked before the examination.) Full regulations are in the JCQ Regulations and Guidance booklet.

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Rain Stopped Play

Introduction

This article concerns the use of mathematical procedures to try to ensure a fair outcome in cricket matches where bad weather has reduced the time available for the match to be completed.

Cricket is a bat-and-ball game involving two teams, referred to here as Team 1 and Team 2, who take it in turn to try to score the most ‘runs’. Runs in cricket correspond to points or goals in other games. 5

In the first part of a match, Team 1 is the ‘batting’ team trying to score runs while Team 2 is the ‘fielding’ team trying to prevent runs being scored. In the second part, the teams exchange roles so that Team 2 becomes the batting team and Team 1 becomes the fielding team. Each part of the game is called an ‘innings’, so that the first part of a game is Team 1’s innings and the second part is Team 2’s innings. 10

During an innings, a player from the fielding team (the ‘bowler’) bowls the ball to a player from the batting team (the ‘batsman’). Each time the ball is bowled, the batsman has the opportunity of hitting the ball in such a way as to be able to score runs, while the bowler has a chance of dismissing the batsman. A batsman who is dismissed can take no further part in the team’s innings, and the batting team is said to have ‘lost a wicket’. At each loss of a wicket the dismissed batsman is replaced by another member of the batting team, until the team’s 10th wicket is lost. At this point no more replacement batsmen are allowed, and the innings of the batting team is complete. 15

Limited Overs Cricket

In the form of cricket discussed in this article, the length of a team’s innings is also limited by the number of times that the fielding team is allowed to bowl the ball. This is expressed as a number of ‘overs’, where an over refers to a particular bowler bowling the ball six times in succession. (This is usually referred to as the bowler ‘bowling six balls’, although the same actual cricket ball is in fact used each time.) So in a ‘50-over’ match, for example, each team’s innings can have at most $50 \times 6 = 300$ balls bowled. An innings will be shorter if a team loses all its 10 wickets before the maximum number of allowed overs have been bowled. 20 25

Matches in limited overs cricket have at most 50 overs per innings, but some matches are shorter, for example 45 or 40 overs per innings.

When bad weather interrupts the innings of either team it may be necessary to reduce the total number of overs that are available in the match, and so a revised target for the number of runs that Team 2 needs to score in order to win may be required. The number of runs that a team has scored and the number of wickets that it has lost when interruptions occur are both factors that need to be taken into account when the match resumes. These two factors are presented in a standard notation so that $127/4$, for example, indicates that a team has scored 127 runs for the loss of 4 wickets. 30

Various different methods have been used to tackle this issue of setting a revised target when bad weather interrupts a match. One of these methods is the Average Run-Rate method. 35

Average Run-Rate (ARR) method

The Average Run-Rate (ARR) method was used at the start of limited overs cricket in the 1960s to set a target number of runs for Team 2 if bad weather reduced the time available for a game.

For example, if Team 1 scores 247 runs in 50 overs, then this gives an ARR of

$$\frac{247}{50} = 4.94 \text{ runs per over.} \quad 40$$

If Team 2's innings is reduced to 33 overs, the target score for Team 2 would be $4.94 \times 33 = 163.02$.

As this is a non-integer value Team 2 would require 164 runs to win. If the target score had been an integer n then Team 2 would have required $n + 1$ runs to win.

It was generally felt that this method was not completely fair, as achieving a given average run rate for a smaller number of overs, without regard to the number of wickets that could be lost, was an easier task for Team 2. However, the method's simplicity, and a lack of any viable alternative, meant that the ARR method was used until the early 1990s. 45

One method that was tried as an alternative to the ARR method was the Most Productive Overs (MPO) method. In this method, Team 2 has a harder task as their target is equivalent to the total scored by Team 1 in its n highest scoring overs, where n is the number of overs available to Team 2. 50

The deficiency of this method was highlighted in a match between England and South Africa in 1992. In the closing stages of the match South Africa required 22 runs to win with 13 balls remaining. Rain stopped play for 12 minutes which meant that only 1 further ball could be bowled. The revised target under the MPO method, was 21 runs from 1 ball, which was an impossible target given that the maximum score from one ball is generally 6 runs. 55

After this match the International Cricket Council (ICC) appealed to mathematicians to come up with something better. Two British mathematicians, Frank Duckworth and Tony Lewis, took up this challenge and their method, known as the Duckworth-Lewis (D/L) method, was first used in 1997.

Duckworth-Lewis (D/L) method

The D/L method is based on the idea that, in building up its score of runs, a team has two ‘resources’ available: the number of wickets remaining and the number of overs to be bowled. A team’s available resources decrease as its innings progresses as there are progressively fewer overs left to be bowled and wickets will usually be lost also. 60

Let R_1 be the total resources used by Team 1 in their innings and let R_2 be the total resources available to Team 2 in their innings. R_1 and R_2 take values between 0 and 100 and are the percentages of the total resources available to a team at the start of a 50-over match. 65

If S_1 is the number of runs scored by Team 1, then the D/L method sets the target number of runs required by Team 2 to be S_2 , where

$$S_2 = S_1 \times \frac{R_2}{R_1} \text{ when } R_2 < R_1, \quad (1)$$

$$S_2 = S_1 \text{ when } R_2 = R_1, \quad (2) \quad 70$$

$$S_2 = S_1 + \left(\frac{R_2 - R_1}{100} \right) \times G50 \text{ when } R_2 > R_1, \text{ where } G50 \text{ is a constant.} \quad (3)$$

Equation (1) deals with the case when Team 2’s resources are less than Team 1’s and the target for Team 2 is reduced in proportion to the resources.

Equation (2) indicates that no change in target is required when resources are equal.

Equation (3) applies when Team 2 has more resources available than Team 1, and their target score is increased by an amount proportional to the extra resources. The constant $G50$ in this equation is the average number of runs scored in a full 50-over innings at the appropriate level of the game, so that the increase in the target for Team 2 is the number of runs that an average team could expect to score with that amount of resources. As of 2014, the value of $G50$ in international matches is 245, and unless otherwise stated this value will be used in calculations in this article. 75 80

As before, if the target score for Team 2 is a non-integer, then Team 2’s target to win is rounded up to the next integer. If the target score for Team 2 is an integer n , then Team 2 needs to score $n + 1$ runs to win.

For example, if Team 2 has only 70% of the resources that were available to Team 1, and Team 1 scored 273 then

$$S_2 = 273 \times 0.7 = 191.1, \quad 85$$

so Team 2’s target to win is 192.

A single table gives the resources remaining at any stage of an innings for any combination of overs remaining and wickets lost. The resources are expressed as percentages of the resources available at the start of a full 50-over innings. An abbreviated version of the D/L resource table is shown in Table 1.

Overs left	Wickets lost									
	0	1	2	3	4	5	6	7	8	9
50	100	93.4	85.1	74.9	62.7	49.0	34.9	22.0	11.9	4.7
49	99.1	92.6	84.5	74.4	62.5	48.9	34.9	22.0	11.9	4.7
48	98.1	91.7	83.8	74.0	62.2	48.8	34.9	22.0	11.9	4.7
47	97.1	90.9	83.2	73.5	61.9	48.6	34.9	22.0	11.9	4.7
46	96.1	90.0	82.5	73.0	61.6	48.5	34.8	22.0	11.9	4.7
45	95.0	89.1	81.8	72.5	61.3	48.4	34.8	22.0	11.9	4.7
44	93.9	88.2	81.0	72.0	61.0	48.3	34.8	22.0	11.9	4.7
43	92.8	87.3	80.3	71.4	60.7	48.1	34.7	22.0	11.9	4.7
42	91.7	86.3	79.5	70.9	60.3	47.9	34.7	22.0	11.9	4.7
41	90.5	85.3	78.7	70.3	59.9	47.8	34.6	22.0	11.9	4.7
40	89.3	84.2	77.8	69.6	59.5	47.6	34.6	22.0	11.9	4.7
35	82.7	78.5	73.0	66.0	57.2	46.4	34.2	21.9	11.9	4.7
30	75.1	71.8	67.3	61.6	54.1	44.7	33.6	21.8	11.9	4.7
25	66.5	63.9	60.5	56.0	50.0	42.2	32.6	21.6	11.9	4.7
24	64.6	62.2	59.0	54.7	49.0	41.6	32.3	21.6	11.9	4.7
23	62.7	60.4	57.4	53.4	48.0	40.9	32.0	21.5	11.9	4.7
22	60.7	58.6	55.8	52.0	47.0	40.2	31.6	21.4	11.9	4.7
21	58.7	56.7	54.1	50.6	45.8	39.4	31.2	21.3	11.9	4.7
20	56.6	54.8	52.4	49.1	44.6	38.6	30.8	21.2	11.9	4.7
19	54.4	52.8	50.5	47.5	43.4	37.7	30.3	21.1	11.9	4.7
18	52.2	50.7	48.6	45.9	42.0	36.8	29.8	20.9	11.9	4.7
17	49.9	48.5	46.7	44.1	40.6	35.8	29.2	20.7	11.9	4.7
16	47.6	46.3	44.7	42.3	39.1	34.7	28.5	20.5	11.8	4.7
15	45.2	44.1	42.6	40.5	37.6	33.5	27.8	20.2	11.8	4.7
10	32.1	31.6	30.8	29.8	28.3	26.1	22.8	17.9	11.4	4.7
5	17.2	17.0	16.8	16.5	16.1	15.4	14.3	12.5	9.4	4.6

Table 1

As can be seen from the table, the resources available when overs are lost for a team batting second are not simply proportional to the number of overs lost. For example, in a 50-over match where Team 2 loses 10 overs at the start of their innings, the table gives 89.3 as the (percentage) resources for a team with 40 overs remaining and no wickets yet lost. This is significantly greater than the 80% that would correspond to the ARR method. So the D/L method can set a higher target for Team 2 than the ARR method, thus attempting to remove the unfairness shown by ARR to Team 1.

90

95

Table 1 is used for all matches, not only full 50-over ones. In a 40-over match, for example, Team 1 starts its innings with a resource value of 89.3, just as though 10 overs had been lost at the start of a full-length match.

The values in Table 1 were calculated from a mathematical formula giving the average number of runs, z , obtainable when a team has w wickets remaining with u overs left. The original formula developed was 100

$$z = z_0(1 - e^{-bu}), \quad (4)$$

though there have been some subsequent modifications. In equation (4), both b and z_0 are functions of w , though the actual definitions of these functions have never been revealed, due to commercial confidentiality.

Details of the how the D/L method works are shown in the following examples, which illustrate some of the ways in which interruptions can occur. 105

Team 1's innings complete; Team 2's innings cut short

45 overs per innings
 Team 1 score 298 in their innings
 Rain reduces Team 2's innings to 35 overs

110

In this scenario Team 1 started their innings with 45 overs and no wickets lost so according to Table 1, Team 1 began with 95.0% of the resources they would have had in a 50-over innings. Team 1 lost no resources due to the interruption so $R1 = 95.0$. The value of $S1$ is 298.

Team 2 start their innings with 35 overs and no wickets lost and so according to Table 1, $R2 = 82.7$.

$R2 < R1$, so using equation (1)

115

$$S2 = 298 \times \frac{82.7}{95.0} = 259.416\dots$$

and Team 2 are therefore set a target of 260 runs to win.

Team 1's innings cut short; Team 2's innings complete

50 overs per innings
 Team 1 score 165/4 in 35 overs
 It rains, leaving only enough time for a further 35 overs in the match
 Team 2 must be set a target for their 35 overs

120

In this scenario Team 1 started their innings with 100% of their resources. When the rain came Team 1 had 15 overs remaining and had lost four wickets. According to Table 1 they had 37.6% resources available at that point, so they had only used up 62.4% of the original amount in scoring their 165 runs. The values for Team 1 are therefore $R1 = 62.4$ and $S1 = 165$. 125

Team 2 started their innings with only 35 overs and no wickets lost and so according to Table 1 they had 82.7% of the resources they would have had if they had their full complement of 50 overs so $R2 = 82.7$. In this case $R2 > R1$ and so equation (3) is used to set the target for Team 2. Using $G50 = 245$ gives

$$S2 = 165 + 245 \left(\frac{82.7 - 62.4}{100} \right) = 214.735. \quad 130$$

So Team 2 are set a target of 215 runs to win.

Team 1's innings interrupted; Team 2's innings completed**45 overs per innings**

Team 1 scores 172/5 in 30 overs and then rain causes the match to be reduced to 40 overs per team

Team 1 scores 220/7 in 40 overs

Team 2 must be set a target for their 40 overs

135

In this scenario Team 1 started their innings with 95.0% of their resources. When the rain came Team 1 had 15 overs left and had lost 5 wickets. Therefore Team 1 had 33.5% resources available at that point. When play resumed Team 1 had only 10 overs left, still with 5 wickets lost, and so they now only had 26.1% resources available. So the rain delay resulted in a loss of resources of $33.5 - 26.1 = 7.4\%$. Therefore the value of R_1 , the resources used by Team 1, is $95.0 - 7.4 = 87.6$.

140

Team 2 started their innings with 40 overs and no wickets lost and so $R_2 = 89.3$. In this case $R_2 > R_1$ and so the score, S_2 , required by Team 2 as given by equation (3) is

$$S_2 = 220 + 245 \left(\frac{89.3 - 87.6}{100} \right) = 224.165.$$

So Team 2 are set a target of 225 runs to win.

145

Along with the cases illustrated above, the D/L method can be used in a range of other scenarios. For example, matches being prematurely abandoned and multiple interruptions in either team's innings can be dealt with. There are also cases where the D/L method sets a target that requires Team 2 to score *fewer* runs than Team 1 in the same number of overs. This seems surprising at first sight, but is in fact an indication that Team 1 performed rather poorly in their innings.

150

In Conclusion

The version of the D/L method seen in this article is relatively transparent and easy to implement and was used widely in all forms of the game until 2003 but it unfortunately had a flaw when handling very high first innings scores. Over the last decade or so the D/L method has evolved so that it requires a computer to perform the calculations required to deal with this flaw but the version seen in this article is still used in most school and club games where computers are not widely available.

155

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POST-EXAM CORRECTION

A2 GCE Mathematics (MEI) 4754/01B Applications of Advanced Mathematics (C4) Paper B: Comprehension

Friday 22 June 2018 –Morning

If you wish to use the published question paper as practice material, please make the following correction.

Turn to **page 6** of the **insert** and look at line **105**.

Cross out the first 'the'.

The sentence should now read:

Details of how the D/L method works are shown in the following examples, which illustrate some of the ways in which interruptions can occur.

If you have any queries about this update, please call our Customer Contact Centre on 01223 553998 or email general.qualifications@ocr.org.uk



Oxford Cambridge and RSA

Friday 22 June 2018 – Morning

A2 GCE MATHEMATICS (MEI)

4754/01B Applications of Advanced Mathematics (C4) Paper B: Comprehension

QUESTION PAPER

Candidates answer on the Question Paper.

OCR supplied materials:

- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: Up to 1 hour



Candidate forename		Candidate surname	
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Centre number						Candidate number				
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INSTRUCTIONS TO CANDIDATES

- The Insert will be found inside this document.
- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Write your answer to each question in the space provided. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- The Insert contains the text for use with the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.
- This document consists of **8** pages. Any blank pages are indicated.

- 1 In a 40-over match, Team 1 scored 183/8 in their 40 overs. Team 2, in the first 23 overs of their innings, reached 102/4 when rain caused a delay. This delay meant that only 5 more overs were available to Team 2. Using the ARR method, calculate how many runs Team 2 had to score in these 5 overs in order to win the match. [3]

1	

PLEASE DO NOT WRITE IN THIS SPACE

- 2 In line 94 the article says that the D/L method can set a higher target for Team 2 than the ARR method would have set.

In a 50-over match Team 1 scores 239 runs. Team 2 only have 40 overs available. Calculate how many more runs Team 2 need to score to win the match if the D/L method, rather than the ARR method, is used. [3]

2	

PLEASE DO NOT WRITE IN THIS SPACE

3 Rewrite equation (4) in line 101 to make the b the subject.

[2]

3	

PLEASE DO NOT WRITE IN THIS SPACE

- 4 The article, in line 147, says that the D/L method can be used to deal with a match in which there are multiple interruptions in either team's innings.

In 2003, a 50-over match between two teams took place and before play began the match was reduced to 46 overs each.

Rain stopped play when Team 1 reached 123/2 from 25 overs. At the restart both innings were reduced to 43 overs.

Rain stopped play again when Team 1 had reached 150/3 from 33 overs, and at the restart both innings were reduced further to 38 overs.

Team 1 finished on 185/3 from their 38 overs.

- (i) Complete the final column of the table below. [4]
- (ii) Calculate the target score to win for Team 2 given that in 2003 the value of G50 was 235. [2]

4(i)		Overs left and wickets remaining	Resource as a percentage
	Total resource available to Team 1 at the start	46 overs left, 10 wickets remaining	
	Total resource remaining to Team 1 at the first interruption	21 overs left, 8 wickets remaining	
	Total resource remaining to Team 1 at the restart	18 overs left, 8 wickets remaining	
	Total resource lost by first interruption		
	Total resource remaining to Team 1 at the second interruption	10 overs left, 7 wickets remaining	
	Total resource remaining to Team 1 at the second restart	5 overs left, 7 wickets remaining	
	Total resource lost by second interruption		
	Total resource available to Team 1		
	Total resource available to Team 2	38 overs left, 10 wickets remaining	86.7
4(ii)			

5 In lines 148 and 149 the article says

‘There are also cases where the D/L method sets a target that requires Team 2 to score *fewer* runs than Team 1 in the same number of overs.’

By calculating the target score required by Team 2 to win, show that the above statement is true in the following scenario.

50 overs per innings
Team 1 scores 110/8 in 35 overs
Rain causes Team 1’s innings to be terminated and Team 2 have 35 overs for their innings

[4]

5	

END OF QUESTION PAPER

GCE

Mathematics (MEI)

Unit **4754B**: Applications of Advanced Mathematics: Comprehension
Paper B: Comprehension

Advanced GCE

Mark Scheme for June 2018

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g Rules for replaced work
- If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.
- If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

- h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1			$\frac{183}{40} (= 4.575)$	B1	Correct calculation of the ARR for Team 1
			$4.575 \times 28 = 128.1$ (so 129 to win)	M1	Complete method (so must be multiplying their ARR by 28) to find the total number of runs required by Team 2
			$129 - 102 = 27$ (runs required in the last 5 overs)	A1	Allow for $128.1 - 102 = 26.1$ therefore 27
				[3]	

Question	Answer	Marks	Guidance
2	$\text{D/L: } 239 \times \frac{89.3}{100} = 213.427$ $\text{ARR: } 239 \times \frac{40}{50} = 191.2$ $214 - 192 = 22 \text{ runs}$	<p>M1</p> <p>A1</p> <p>A1</p>	<p>Correct calculation with correct values for the runs required by either the D/L or ARR method</p> <p>Both values correct to at least 1 decimal place</p> <p>A0 if $213 - 191 = 22$</p>
		[3]	
3	$z = z_0(1 - e^{-bu})$ $e^{-bu} = 1 - \frac{z}{z_0}$ $-bu = \ln\left(1 - \frac{z}{z_0}\right)$ $b = -\frac{1}{u} \ln\left(1 - \frac{z}{z_0}\right)$	<p>B1</p> <p>B1</p>	<p>Correctly re-arranging to make e^{-bu} the subject</p> <p>Or correct equivalent (dependent on previous B mark) – bracket must be present</p>
		[2]	

Question		Answer	Marks	Guidance
4	(i)	Resource as a percentage	B2	B2 for 96.1, 54.1, 48.6, 29.8 and 16.5 B1 for any three correct
		96.1		
		54.1		
		48.6		
		5.5 ('54.1' – '48.6')		
		29.8	B1ft	B1ft for 5.5 (their 54.1 – their 48.6) and 13.3 (their 29.8 – their 16.5)
		16.5		
		13.3 ('29.8' – '16.5')		
		77.3 ('96.1' – '5.5' – '13.3')	B1ft	B1ft for 77.3 (their 96.1 – their 5.5 – their 13.3)
		86.7		
			[4]	
4	(ii)	$S2 = 185 + 235 \left(\frac{86.7 - 77.3}{100} \right)$	M1	Correct method for calculating S2 using their 77.3 – note that their 77.3 must be strictly less than 86.7 for this mark – M0 if no 77.3 value stated in the table in part (i) – for this mark all other values used must be correct
		= 207.09 ⇒ 208 target	A1	cao
			[2]	

Question	Answer	Marks	Guidance
5	<p>15 overs remaining and 8 wickets lost \Rightarrow 11.8% resource remaining</p> <p>so $R1 = 88.2$ and $R2 = 82.7$</p> $S2 = 110 \times \frac{82.7}{88.2} (=103.14\dots)$ <p>Target of $104 < 110$ so it is correct that Team 2 is not set an enhanced target</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>E1</p>	<p>Implied by seeing $R1 = 88.2$ (or just 88.2)</p> <p>Seen or implied (could be seen in a formula for S2)</p> <p>M1 for $110 \times \frac{82.7}{88.2}$ only (so with their 88.2 but everything else must be correct)</p> <p>Must be using 88.2 (note that a resource of 11.9% will give 104 too but this only scores M1) – for this mark we must have 104 + a conclusion in context (e.g. comparing 104 with 110, stating that the target is less, etc.)</p>
		[4]	

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AS/A LEVEL GCE

Examiners' report

MATHEMATICS (MEI)

3895-3898, 7895-7898

4754/01B Summer 2018 series

Version 1

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Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper 4754/01B series overview

Applications of Advanced Mathematics (core 4) 4754 is the fourth mandatory component of 7895 A Level Mathematics (MEI). This component is made up of an examination paper and this separate comprehension task. This is the final assessment series for this specification, although there is a resit opportunity in summer 2019.

The rationale for the comprehension paper is to assess the ability of candidates to read and comprehend a mathematical argument or an example of the application of mathematics in a specific context and then demonstrate their understanding through undertaking further calculations and commenting on the results in that context.

The comprehension paper this year concerned the mathematical procedures that can be employed to try to ensure a fair outcome in cricket matches when bad weather reduces the time available for the match to be completed. Whilst using a sporting context in a question paper has the potential to introduce bias we take great care to ensure that the questions asked are about the mathematics and do not require any background knowledge of the sport.

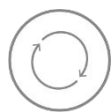
The vast majority of candidates dealt with the demands of this paper well and good marks were scored here.



In order to prepare for this assessment candidates need to read the passage carefully – not skim read as they might a novel, or to rush straight to the questions. It is recommended that candidates read the passage in its entirety first, then read the questions before rereading the passage to identify the relevant information for each question and making notes in the text as appropriate.

There was a minor typo in line 105 of the comprehension insert, this did not affect candidates' ability to answer the question.

Key



AfL

Guidance to offer for future teaching and learning practice.

Question 1

- 1 In a 40-over match, Team 1 scored 183/8 in their 40 overs. Team 2, in the first 23 overs of their innings, reached 102/4 when rain caused a delay. This delay meant that only 5 more overs were available to Team 2. Using the ARR method, calculate how many runs Team 2 had to score in these 5 overs in order to win the match. [3]

The vast majority of candidates correctly calculated the ARR for Team 1 by considering $\frac{183}{40}$. However, many then went on to incorrectly multiply this by 5 to give 22.875; concluding that 23 runs were required. The correct method was to multiply the ARR by 28 (the total number of overs available to Team 2) to give a rounded value of 129 and then to consider $129 - 102$ which gives the 27 runs required in the last 5 overs.

Question 2

- 2 In line 94 the article says that the D/L method can set a higher target for Team 2 than the ARR method would have set.

In a 50-over match Team 1 scores 239 runs. Team 2 only have 40 overs available. Calculate how many more runs Team 2 need to score to win the match if the D/L method, rather than the ARR method, is used. [3]

While the vast majority of candidates scored at least one mark for correctly using one of the methods provided in the passage, many believed that both methods gave the same answer. Furthermore, some candidates only calculated the runs required by one of the two methods. Of those candidates who correctly obtained 213.427 for D/L and 191.2 for ARR nearly all correctly rounded up to 214 and 192 and then went on to calculate the required difference.

Question 3

- 3 Rewrite equation (4) in line 101 to make the b the subject. [2]

Most candidates in this question correctly re-arranged to make e^{-bu} the subject and then successfully went on to take logs (in an attempt to make b the subject).



A number of candidates partially re-arranged and then wrote that

$$-e^{-bu} = \frac{z}{z_0} - 1 \Rightarrow bu = \ln\left(\frac{z}{z_0} - 1\right).$$

Other candidates started this question by first expanding the brackets and then, mistakenly, took logs of each term, for example, writing $z = z_0 - z_0 e^{-bu} \Rightarrow \ln z = \ln z_0 - \ln(z_0 e^{-bu})$



Misconception

Question 4(i)

- 4 The article, in line 147, says that the D/L method can be used to deal with a match in which there are multiple interruptions in either team's innings.

In 2003, a 50-over match between two teams took place and before play began the match was reduced to 46 overs each.

Rain stopped play when Team 1 reached 123/2 from 25 overs. At the restart both innings were reduced to 43 overs.

Rain stopped play again when Team 1 had reached 150/3 from 33 overs, and at the restart both innings were reduced further to 38 overs.

Team 1 finished on 185/3 from their 38 overs.

- (i) Complete the final column of the table below. [4]

4(i)	Overs left and wickets remaining	Resource as a percentage
Total resource available to Team 1 at the start	46 overs left, 10 wickets remaining	
Total resource remaining to Team 1 at the first interruption	21 overs left, 8 wickets remaining	
Total resource remaining to Team 1 at the restart	18 overs left, 8 wickets remaining	
Total resource lost by first interruption		
Total resource remaining to Team 1 at the second interruption	10 overs left, 7 wickets remaining	
Total resource remaining to Team 1 at the second restart	5 overs left, 7 wickets remaining	
Total resource lost by second interruption		
Total resource available to Team 1		
Total resource available to Team 2	38 overs left, 10 wickets remaining	86.7

The majority of candidates scored at least the first two marks for correctly filling in the table with 96.1, 54.1, 48.6, 29.8 and 16.5 for the first, second, third, fifth and sixth values respectively. The values for the 'Total resource lost by first/second interruption' seemed to be the most demanding for candidates even though in both cases these were simply the difference of the two values given directly above these gaps. The vast majority of candidates either correctly calculated the Total resource to Team 1 as 77.3 or scored this mark on the follow through from their (earlier incorrect) values.

Question 4(ii)

- (ii) Calculate the target score to win for Team 2 given that in 2003 the value of G50 was 235. [2]

This part was answered well with most candidates scoring at least the method mark for using the correct formula $S2 = S1 + \left(\frac{R2 - R1}{100}\right) \times G50$ with $S1 = 185$, $R2 = 86.7$, $G50 = 235$ and their value from part 4(i) for $R1$. A number of candidates attempted to use this formula when they had a value of $R1$ greater $R2$. Of those candidates who obtained the correct value of 207.09 most correctly rounded up (rather than down) to obtain the required 208.

Question 5

- 5 In lines 148 and 149 the article says

‘There are also cases where the D/L method sets a target that requires Team 2 to score *fewer* runs than Team 1 in the same number of overs.’

By calculating the target score required by Team 2 to win, show that the above statement is true in the following scenario.

50 overs per innings

Team 1 scores 110/8 in 35 overs

Rain causes Team 1's innings to be terminated and Team 2 have 35 overs for their innings

[4]

The most common error in this part was to state that the resource remaining for Team 1 was 11.9% (which examiners can only assume came from reading across the row for 35 overs left in Table 1) and not the correct 11.8% (which is the value in Table 1 when reading across from the correct row of 15 overs remaining). Although a small number of candidates attempted to use the formula $S2 = S1 + \left(\frac{R2 - R1}{100}\right) \times G50$ to calculate the target score for Team 2 most correctly used $S2 = S1 \times \frac{R2}{R1}$ with $R2 = 82.7$, although a number of candidates thought that $R1$ was 100. Of those that correctly found that Team 2 score to win was 104 many did not give a suitable conclusion regarding the fact the D/L method in this scenario sets a target for Team 2 that required them scoring fewer runs than Team 1 scored.

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Question 4 (i)

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Cambridge
Assessment



001

Unit level raw mark and UMS grade boundaries June 2018 series

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AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

AS & Advanced GCE Mathematics				Max Mark	a	b	c	d	e	u
4721	01	C1 Core mathematics 1 (AS)	Raw	72	61	55	50	45	40	0
			UMS	100	80	70	60	50	40	0
4722	01	C2 Core mathematics 2 (AS)	Raw	72	55	49	43	37	31	0
			UMS	100	80	70	60	50	40	0
4723	01	C3 Core mathematics 3 (A2)	Raw	72	55	48	41	34	28	0
			UMS	100	80	70	60	50	40	0
4724	01	C4 Core mathematics 4 (A2)	Raw	72	54	47	40	34	28	0
			UMS	100	80	70	60	50	40	0
4725	01	FP1 Further pure mathematics 1 (AS)	Raw	72	56	50	45	40	35	0
			UMS	100	80	70	60	50	40	0
4726	01	FP2 Further pure mathematics 2 (A2)	Raw	72	59	53	47	41	35	0
			UMS	100	80	70	60	50	40	0
4727	01	FP3 Further pure mathematics 3 (A2)	Raw	72	47	41	36	31	26	0
			UMS	100	80	70	60	50	40	0
4728	01	M1 Mechanics 1 (AS)	Raw	72	60	51	42	34	26	0
			UMS	100	80	70	60	50	40	0
4729	01	M2 Mechanics 2 (A2)	Raw	72	53	46	39	32	26	0
			UMS	100	80	70	60	50	40	0
4730	01	M3 Mechanics 3 (A2)	Raw	72	50	42	34	27	20	0
			UMS	100	80	70	60	50	40	0
4731	01	M4 Mechanics 4 (A2)	Raw	72	59	53	47	42	37	0
			UMS	100	80	70	60	50	40	0
4732	01	S1 – Probability and statistics 1 (AS)	Raw	72	57	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4733	01	S2 – Probability and statistics 2 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4734	01	S3 – Probability and statistics 3 (A2)	Raw	72	59	50	41	32	24	0
			UMS	100	80	70	60	50	40	0
4735	01	S4 – Probability and statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4736	01	D1 – Decision mathematics 1 (AS)	Raw	72	55	48	42	36	30	0
			UMS	100	80	70	60	50	40	0
4737	01	D2 – Decision mathematics 2 (A2)	Raw	72	58	53	48	44	40	0
			UMS	100	80	70	60	50	40	0

AS & Advanced GCE Mathematics (MEI)			Max Mark	a	b	c	d	e	u	
4751	01	C1 – Introduction to advanced mathematics (AS)	Raw	72	60	55	50	45	40	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – Concepts for advanced mathematics (AS)	Raw	72	53	47	41	36	31	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) Methods for Advanced Mathematics (A2): Written Paper	Raw	72	61	56	51	46	40	0
4753	02	(C3) Methods for Advanced Mathematics (A2): Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) Methods for Advanced Mathematics (A2): Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – Applications of advanced mathematics (A2)	Raw	90	63	56	49	43	37	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – Further concepts for advanced mathematics (AS)	Raw	72	55	51	47	43	40	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – Further methods for advanced mathematics (A2)	Raw	72	48	42	36	31	26	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – Further applications of advanced mathematics (A2)	Raw	72	63	56	49	42	35	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) Differential Equations (A2): Written Paper	Raw	72	61	54	48	42	35	0
4758	02	(DE) Differential Equations (A2): Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) Differential Equations (A2): Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – Mechanics 1 (AS)	Raw	72	51	44	37	31	25	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – Mechanics 2 (A2)	Raw	72	59	53	47	41	35	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – Mechanics 3 (A2)	Raw	72	61	54	48	42	36	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – Mechanics 4 (A2)	Raw	72	59	51	44	37	30	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – Statistics 1 (AS)	Raw	72	59	53	47	42	37	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – Statistics 2 (A2)	Raw	72	54	47	41	35	29	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – Statistics 3 (A2)	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – Decision mathematics 1 (AS)	Raw	72	50	44	38	32	26	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – Decision mathematics 2 (A2)	Raw	72	55	51	47	43	39	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) Numerical Methods (AS): Written Paper	Raw	72	57	52	48	44	39	0
4776	02	(NM) Numerical Methods (AS): Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) Numerical Methods (AS): Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
			UMS	100	80	70	60	50	40	0

AS GCE Statistics (MEI)			Max Mark	a	b	c	d	e	u
G241	01	Statistics 1 MEI	Raw	72	No entry in June 2018				
			UMS	100	80	70	60	50	40
G242	01	Statistics 2 MEI	Raw	72	No entry in June 2018				
			UMS	100	80	70	60	50	40
G243	01	Statistics 3 MEI	Raw	72	No entry in June 2018				
			UMS	100	80	70	60	50	40

AS GCE Quantitative Methods (MEI)			Max Mark	a	b	c	d	e	u	
G244	01	Introduction to Quantitative Methods (Written Paper)	Raw	72	58	50	43	36	28	0
			UMS	100	80	70	60	50	40	0
G244	02	Introduction to Quantitative Methods (Coursework)	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
G245	01	Statistics 1	Raw	72	61	55	49	43	37	0
			UMS	100	80	70	60	50	40	0
G246	01	Decision Mathematics 1	Raw	72	50	44	38	32	26	0
			UMS	100	80	70	60	50	40	0

Level 3 Certificate, Level 3 Extended Project and FSMQ raw mark grade boundaries June 2018 series

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Level 3 Certificate Mathematics - Quantitative Methods (MEI)

					Max Mark	a	b	c	d	e	u
G244	A	01	Introduction to Quantitative Methods with Coursework (Written Paper)	Raw	72	58	50	43	36	28	0
G244	A	02	Introduction to Quantitative Methods with Coursework (Coursework)	Raw	18	14	12	10	8	7	0
				UMS	100	80	70	60	50	40	0
				Overall	90	72	62	53	44	35	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI)

					Max Mark	a	b	c	d	e	u
H866		01	Introduction to quantitative reasoning	Raw	72	56	49	42	35	28	0
H866		02	Critical maths	Raw	60	44	39	34	29	24	0
*To create the overall boundaries, component 02 is weighted to give marks out of 72				Overall	144	109	96	83	70	57	0

Level 3 Certificate Mathematics - Quantitative Problem Solving (MEI)

					Max Mark	a	b	c	d	e	u
H867		01	Introduction to quantitative reasoning	Raw	72	56	49	42	35	28	0
H867		02	Statistical problem solving	Raw	60	40	36	32	28	24	0
*To create the overall boundaries, component 02 is weighted to give marks out of 72				Overall	144	104	92	80	69	57	0

Advanced Free Standing Mathematics Qualification (FSMQ)

					Max Mark	a	b	c	d	e	u
6993		01	Additional Mathematics	Raw	100	56	50	44	38	33	0

Intermediate Free Standing Mathematics Qualification (FSMQ)

					Max Mark	a	b	c	d	e	u
6989		01	Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0